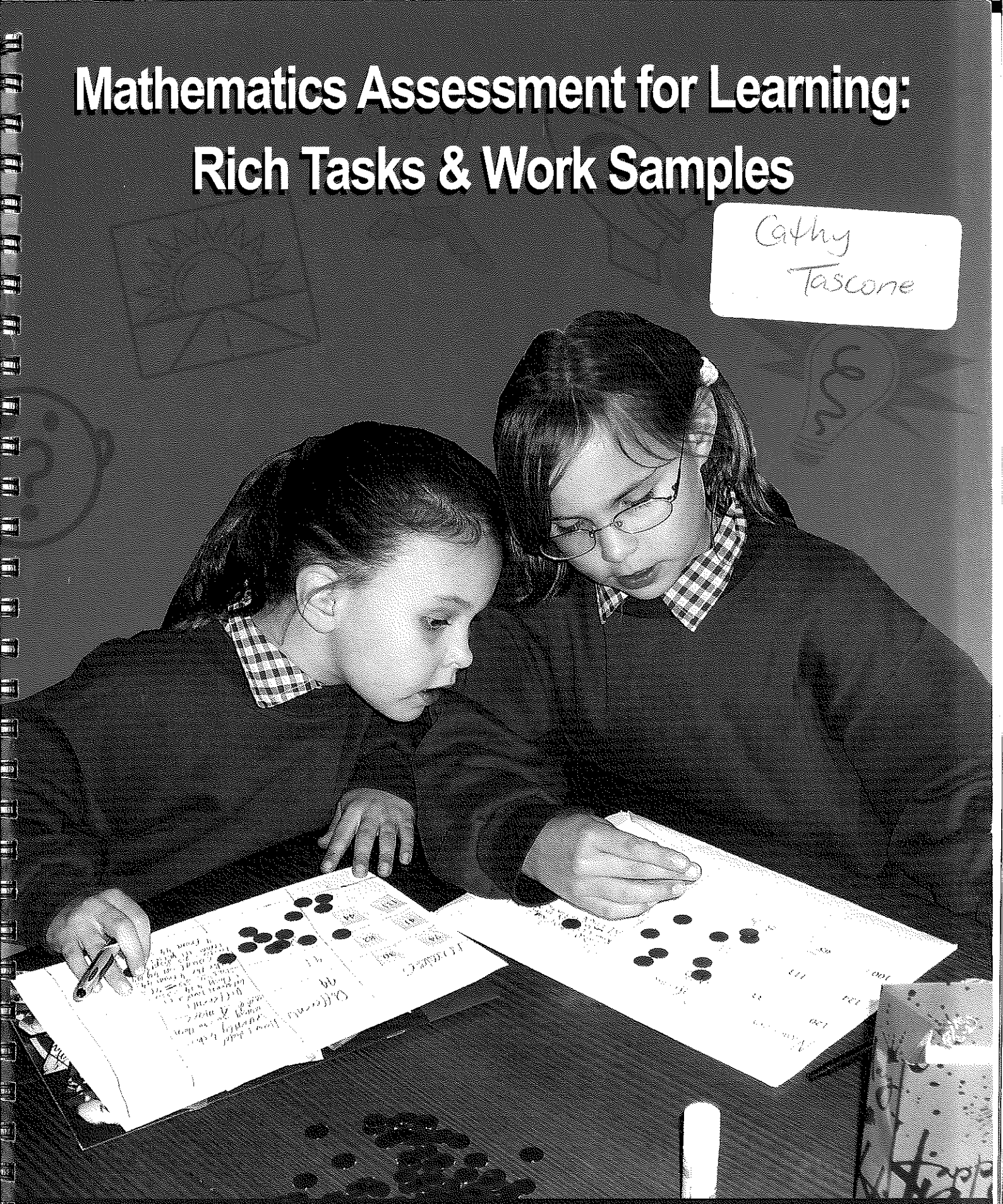
Mathematics Assessment for Learning: Rich Tasks & Work Samples

Cathy Tascone



Ann Downton Rose Knight Doug Clarke Gerard Lewis

Junior Primary to Junior Secondary

TASK 19

Everything About My Fraction

Equipment required:
- Task sheet

TASK DESCRIPTION:

Students choose a fraction and complete the following tasks:
- Write down everything you know about your fraction
- Write down the decimal equivalent of your fraction
- Represent your fraction on a number line
- Find three other fractions equivalent to your fraction
- Represent your fraction as part of a metre
- Cut a piece of wool or string that is this long
- Record the length of the wool in centimetres

MATHEMATICAL CONTENT:

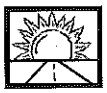
- Concept of a fraction
- Multiple representations of a given fraction

SNIPPETS FROM THE CLASSROOM

One of the things which makes fractions so difficult for many students is that there are many "subconstructs" of fractions (part/whole, division, measure, ratio, rate, etc.), and many representations.
One weakness of this task is that the particular fraction which the student chooses to use or is given by the teacher can greatly affect the level of difficulty. For example, $\frac{1}{2}$ can be more readily converted to a decimal or placed on a number line than, say, $\frac{2}{3}$, for most students. Teachers commented that students would be better placed to "go beyond" if they were encouraged to represent their fraction in a variety of measurement forms in the last part. Teachers may wish to try this variation.
Not surprisingly, given research on students' representation of fractions on a number line, many students were unable to place their fractions correctly on the number line. A major contributor to this is likely to be their lack of classroom experience with this kind of task.

Scoring Rubric

Year 6 Focus



Goes Beyond

- The student indicates a number of ways in which the fraction could be represented other than those strictly required in the task.

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4 

Task accomplished

- Mathematically correct answers to all questions for the nominated fraction.

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3 

Substantial progress

- One error in understanding is evident in the student's response (e.g., incorrect placement or labelling of the number line, or non-equivalent decimal).

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2 

Some progress

- Several correct responses, but at least two difficulties in understanding are evident.

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1 

Little progress

- Little evidence of any connected understanding of the nominated fraction.

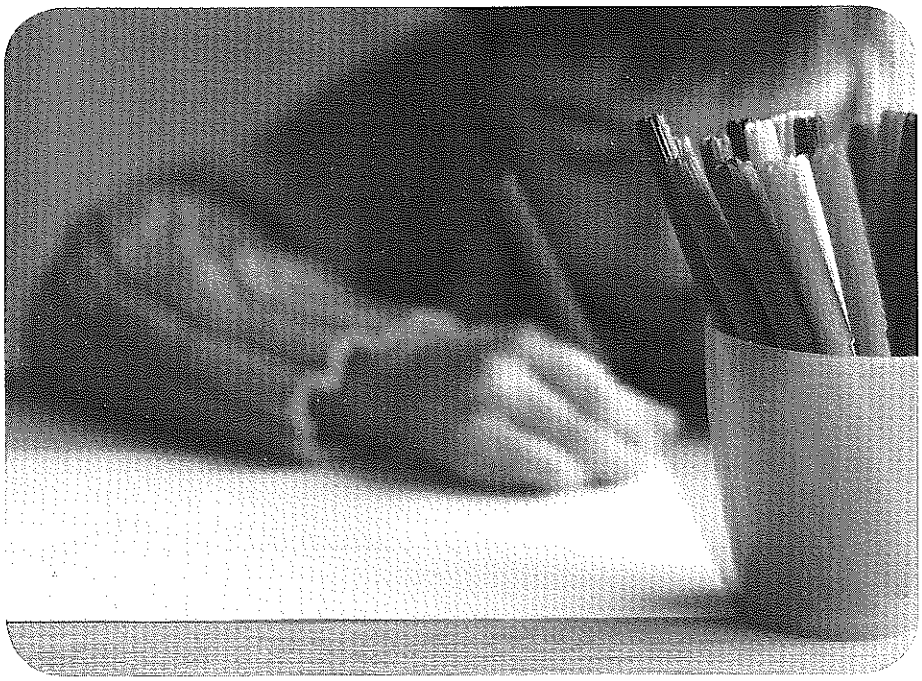
TASK 19 SHEET

Everything About My Fraction

Write down a list of all the fractions you know.
From this list choose your favourite fraction.

Once you have chosen your fraction, answer the following set of questions about "your" fraction.

- Write down everything you know about your fraction.
- Write down the decimal equivalence of your fraction.
- Represent your fraction on a number line.
- Represent the decimal form of your fraction on a number line.
- Find three other fractions equivalent to your fraction.
- Represent your fraction as part of a metre.
- Cut a piece of wool or string that is this long.
- Record the length of the wool in centimetres.



TASK 20

Picturing Multiplication of Fractions

Equipment required:
- Task sheet

TASK DESCRIPTION:

The students are presented with four possible ways in which the product of two mixed numbers (3 ½ and 1½) could be represented. They are asked to choose the one which is correct and explain their thinking.

MATHEMATICAL CONTENT:

- Concept of fraction multiplication
- Visual images of fraction operations

SNIPPETS FROM THE CLASSROOM

It was clear that very few students could conceive of this product of fractions in any other way than as an algorithmic procedure. Many students attempted to justify their choice (where they made one) by multiplying the two relevant fractions expressed as decimals. This led to much confusion about the location of the decimal point in the product. We have observed the difficulty of student teachers with whom we work in representing even ½ x ⅓ or 1 ÷ ⅓ in any meaningful way, so the difficulty evident here is no great surprise. Is this because it is inherently difficult to understand, or is it a reflection of the way in which these topics are typically approached at school? Very few students appeared to have considered the pictures in formulating their response, except in order to check the answer arising from their algorithm.

A teaching point is that students need to be encouraged to move beyond "groups of" as their only way of conceiving of multiplication, as multiplication moves beyond whole numbers. It may be helpful to think about this product as 1 ½ lots of 3 ½, which is well represented by the picture in part B. It will also be useful to discuss the fact that this can also be thought of as 1 lot of 3 ½ and a half a lot of 3 ½ , or, using the commutativity and distributive property, 3 lots of 1 ½ and ½ a lot of 1 ½.

Scoring Rubric

Year 7 Focus

Goes Beyond
- The student provides more than one convincing explanation for why choice B is correct (e.g., indicating how the picture represents 1 ½ lots of 3 ½ as well as 3 ½ lots of 1 ½.).

4 Task accomplished
- Clearly relates the picture to the items which are being multiplied, showing the clear link.

3 Substantial progress
- The student shows by an appropriate calculation that the answer to this product must be 5 ¼ or 5.25, but doesn't relate it to the arrangement of shapes in the picture other than to confirm the answer.

2 Some progress
- Makes the correct choice, but the explanation lacks clarity.

1 Little progress
- Incorrect answer or absence of any meaningful explanation.

TASK 24

Giving Directions

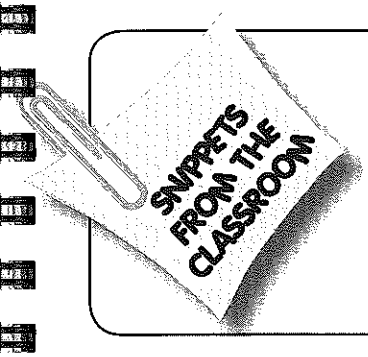
Equipment required:
- Photocopied page from street directory (a variety of pages, two copies each)

TASK DESCRIPTION:

Work with a partner. Each person needs a copy of the same page from a street directory. Choose two places on the map, a starting point and a finishing point. Show your partner your starting point but not your finishing point. Write directions for getting from your starting point to your finishing point. Take turns to read or tell your directions to your partner. Your partner has to follow your directions on their copy of the map. Does your partner end up at the right place? If not, why not? In what ways could you improve your directions?

MATHEMATICAL CONTENT:

- Language of position, location (left, right, north, south, etc.)
- Giving directional instructions
- Visualisation



Although many students reported that their partner ended up at "the right place", the instructions appeared to be ambiguous or incomplete in many cases. Stronger responses included those which used street names and landmarks, and made it clear whether left turn referred to the first or second possible left turn, for example. In order to illustrate the need to be specific, some teachers found it helpful to read aloud some responses, with all students following the instruction on their own copy of the map.

Scoring Rubric

Year 6 Focus

Goes Beyond
- In addition to clear, unambiguous instructions, the work sample provides examples of additional insights such as any likely difficulties which may be encountered in following the directions. It uses scale to calculate relevant distances, or uses insights into slope of hills or accessibility to refine the recommended map.

4 Task accomplished
- Clear and unambiguous instructions with no redundant information.
- Street names and landmarks mentioned as appropriate.
- Recommendations show clear evidence of reflection and analysis of initial instructions.

3 Substantial progress
- Instructions generally clear, but some ambiguity or inadequacy in instructions.
- Suggestions for improvement include awareness of inadequacies of their instructions, indicating evidence of reflection.

2 Some progress
- Instructions lead in the general direction, but considerably lacking in enough clear information as to the starting point, finishing point, or recommended path.
- Suggestions for improvement offer little evidence of reflection.

1 Little progress
- Some directional information, but little evident understanding of what is expected in the task.

TASK 25

Maps for the Commander*

Equipment required:
- Task sheet
- A range of concrete material

TASK DESCRIPTION:

The students are presented with a scenario in which three views of a city, surrounded by a circular wall, are required, prior to an attack on the city. Only two are available however, and the students are challenged to draw the third view, the one from the North-East, explaining their reasoning.

MATHEMATICAL CONTENT:

- Spatial visualisation and orientation
- 2-D representations of 3-D situations

SNIPPETS FROM THE CLASSROOM

Some teachers provided an introduction to this activity for students who had never worked on problems of this kind, by gathering the class around a collection of objects, and discussing what could be seen or not seen from various positions. This proved to be quite a difficult task, but generally speaking, those students who either modelled the situation with materials, or drew a bird's-eye view of the situation, found a reasonable solution. There was some difficulty with students' understanding of compass directions. This is one of those tasks where teacher observation and questioning is a key source of assessment information, as the students are working.

Scoring Rubric

Year 8 Focus

Goes Beyond

- Elaborate method that shows reasoning beyond just achieving the task, including statements of the kind ("the tower must be in the middle from any direction if it is in the middle from two directions, in a circular city").

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4 

Task accomplished

- Drawing shows correct view (three buildings lined up in front of each other: dome first, "pointy" tower behind, square building behind that).
- Clear explanation of method of solution (physical modelling, bird's-eye view or other).

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3 

Substantial progress

- Final solution "close" but not correct, with appropriate strategy used and explained OR
- Correct solution with missing or inadequate explanation.

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2 

Some progress

- Reasonable method but the solution is a long way from the desired one.
- Explanation inadequate or bears little relevance to the method used.

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1 

Little progress

- Ignores key information (e.g., circular wall) or little evidence of understanding how the view varies according to the direction, or draws a "picture" of the objects rather than a view.

TASK 29

* Map reproduced with permission from Pearson Education Australia (from Profiling Mathematics, Level 4, R. Griffiths & M. Clyne)

Travel Plans*

Equipment required:
- Map of journey

TASK DESCRIPTION:

Students plan a journey using a map showing the route from Melbourne to Brisbane on the Newell Highway or any road map. They plan petrol and other stopping places and estimate the time for the complete journey. Explain that they can travel up to 300 kilometres on one tank of petrol. They can work in small groups to discuss the possible petrol stops, average speed, and reasons for and duration of other stops, but they record their explanations and justifications individually.

MATHEMATICAL CONTENT:

- Location—reading and interpreting a map
- Calculating total distance travelled
- Time—calculating durations of time

SNIPPETS FROM THE CLASSROOM

While the students generally enjoyed the task, many of them found the calculation of the total duration of time rather challenging and, as indicated by the work samples, some had little idea as to how to calculate the time travelled per tank of petrol. Many recorded an estimated time for the total journey but did not include any evidence of how they arrived at it.

Scoring Rubric

Year 6 Focus

Goes Beyond
- Calculates the time taken using the amount travelled per hour and includes the total amount of petrol required for the trip.

4 Task accomplished
- Well planned and clearly explained trip with use of the map to calculate the distance travelled before needing to stop for fuel.
- Realistic estimation of duration of time for the total trip, with some evidence of how it was ascertained and correct calculation of total kilometres travelled (1130).

3 Substantial progress
- Clear explanation of trip and evidence of use of the map to plan and calculate distances.
- Evidence of distances travelled and how total distance was calculated, but little evidence of how the duration of the trip was calculated, incomplete, or not completely correct.

2 Some progress
- Explanation of the trip clearly indicates reference to the map for planned stops and some reference to distance travelled to certain towns.
- Time taken to drive between stops and or total kilometres travelled may not be quite accurate or realistic.

1 Little progress
- Recording includes some evidence of use of the map to plan the route and where to stop for breaks, but some are unrealistic.
- Little reference to distance travelled between stops, duration of stops, or estimate of total duration of the journey was given, some recording of total distance travelled was included, but is incorrect.

TASK 33

Granny's Rug

Equipment required:
- Task sheet
- Newspaper, scrap paper, rulers, tape measures, sticky tape.

TASK DESCRIPTION:

Pose the scenario: My granny bought a square rug for her hallway and each side measured one metre. When she got it home, it would not fit in the hallway, so she cut the rug up and joined the pieces together again to make the shape that would fit using all the rug. What might her rug look like now?

Make granny's original rug out of newspaper and work out the area and cut it up to make a rug that might fit. What does it look like and what is the area of the rug?

Draw a diagram of a number of different ways granny's rug might look like now, and write about what you did and what you found out.

MATHEMATICAL CONTENT:

- Area and perimeter

SNIPPETS FROM THE CLASSROOM

Students needed to use the newspaper to construct the different shapes as some found this a very challenging task, especially if they started with diagrams and then had to calculate the area when decimals were involved. Many of the work samples indicated their awareness of the need to create a long rug. They had difficulty in calculating the area or realising that the area would remain constant as the whole piece of paper was used but just arranged differently.

Scoring Rubric *Year 8 Focus*

Goes Beyond

- Generalises that the area of an enclosed shape remains the same when reconfigured, but can result in different perimeters. Similarly, shows an understanding that for a fixed perimeter, a shape can have different areas.

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4 

Task accomplished

- Three or more correct diagrams with clear explanation of the method used, and accompanying calculations are correct.
- Samples indicate an understanding that different configurations of the one shape have the same area but different perimeters.

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3 

Substantial progress

- Diagrams are correct and clearly labelled with measurements and there is a clear explanation of method used, but they haven't included the calculation or the actual area of the new rugs.

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2 

Some progress

- One correct diagram, but the calculation is incorrect or is of the perimeter of the new shape and not the area.

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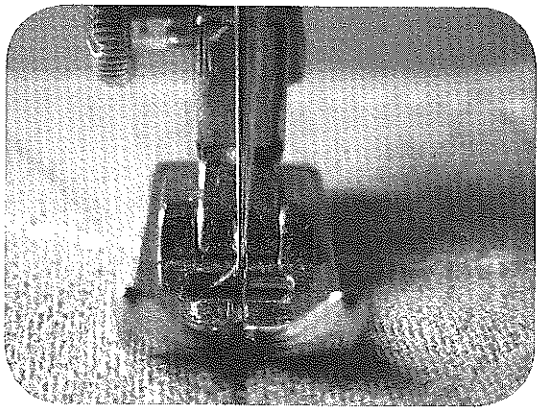
1 

Little progress

- Realises the rug needs to be cut up and reconfigured in some way, but has little idea of how to calculate the area of the new shape. May attend to the perimeter rather than area.
- Not aware that the whole shape needs to be used.

TASK 33 SHEET

Granny's Rug



My granny bought a square rug for her hallway and each side measured one metre. When she got it home it would not fit in the hallway, so she cut the rug up and joined the pieces together again to make the shape that would fit, using all the rug. What might her rug look like now?

Make granny's original rug from newspaper and work out the area. Now cut it up to make a rug that might fit. What does it look like and what is the area of the rug?

Draw a diagram of what granny's rug might look like now and write about what you did and what you found out.

CONGRATULATIONS!

You have won a money prize. Your prize can be:

A one litre milk carton filled with 20c coins

OR

1 kilogram of $1 coins

OR

A line of $2 coins 1 metre long (lying flat and touching)

OR

A square metre of 5c coins (lying flat and touching)

Which will you choose?

Explain how you made an estimate of each one and show clearly any calculations.

TASK 13 SHEET

Helping Bert Divide

Bert did some division questions like this

$$4\overline{)628}\ 157 \qquad 6\overline{)1248}\ 28 \qquad 3\overline{)4815}\ 165$$

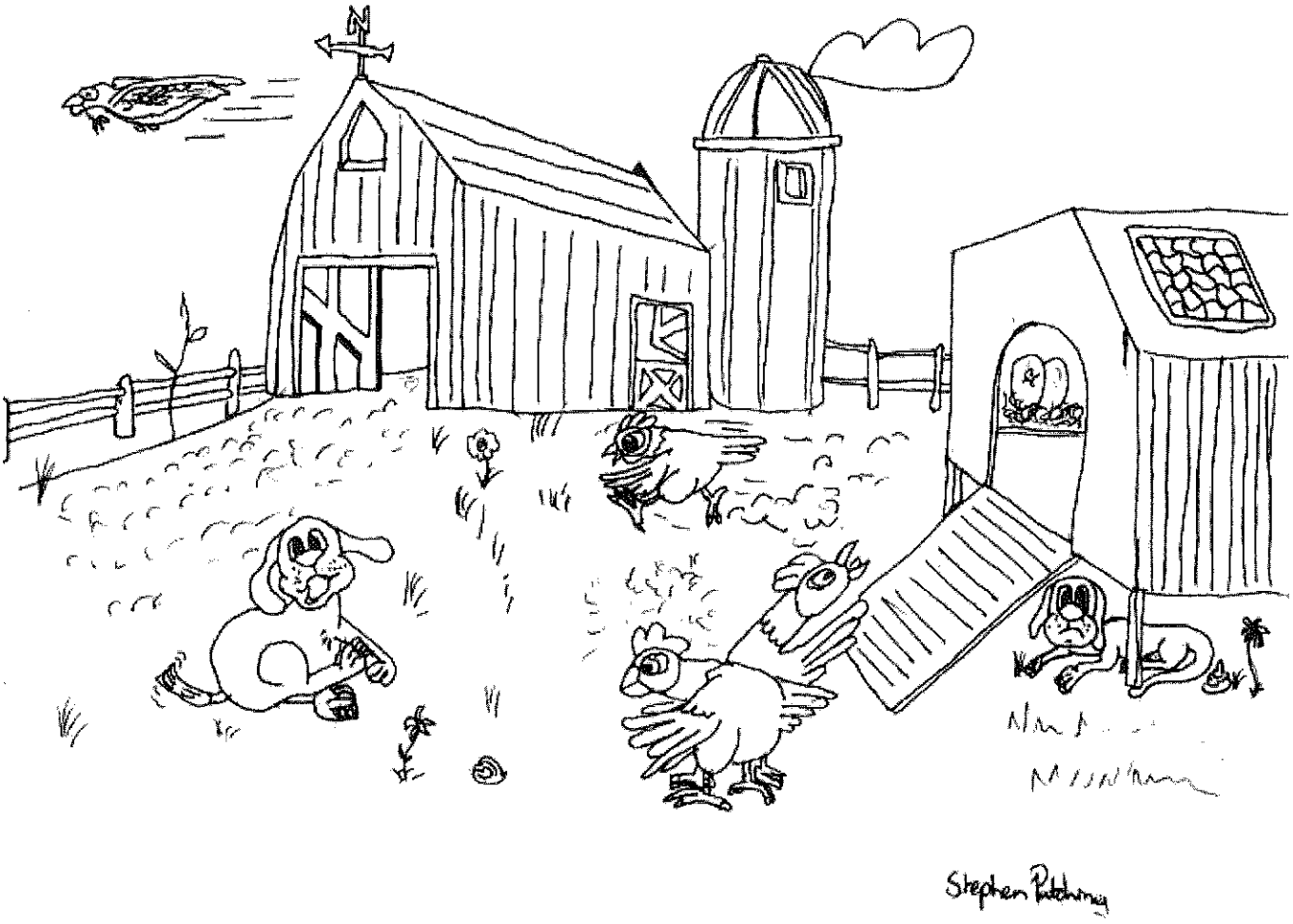
1. If the answer is correct, tick it. If not, write the correct answer.
2. In the box below write down some advice you would give Bert.

3. Write down one question you think Bert would get correct, and write down the answer.

4. Write down some questions which you think Bert might get wrong. Give both the answer Bert might give, and the correct answer.

TASK 14 SHEET Heads and Legs

In my backyard, I have some chooks and some dogs.
Altogether I can count 25 heads and 78 legs. How many dogs do I have?



TASK 15

Mike and His Numbers

Equipment required:
- Newspaper article and task

TASK DESCRIPTION:

Read the newspaper article, and then pose the question, "If you wrote all the numbers from 1 to one million, how many digits would you use altogether?" The answer in the paper (5 878 936) is incorrect, by the way.

MATHEMATICAL CONTENT:

- Number patterns
- Systematic listing of possibilities
- Multiplication of whole numbers (or calculator use)

SNIPPETS FROM THE CLASSROOM

Students were very interested in the story of Mike and his perseverance. They had difficulty conceiving of how long it would take to do such a thing. Some teachers asked students to estimate how long it would take them to write the numbers from 1 to 100. In one Year 5/6 class, answers varied from 30 seconds to two hours! This was a challenging task for most students, with surprising difficulty with the notion that if we wrote the two-digit numbers from 10 to 99, there would be 90 numbers, not 89.

Scoring Rubric

Year 7 Focus

Goes Beyond

- Generalises the task in some way so that it is clear how any task of the form "1 to n (how many digits)" could be solved.

4 

Task accomplished

- Systematic method applied correctly, yielding the answer 5 888 896.

3 

Substantial progress

- Uses an appropriate method quite well, but one or more computations incorrect, or forgets that there is only one 7-digit number (1 000 000).

2 

Some progress

- Commences a systematic approach (e.g., how many one-digit numbers, how many two-digit numbers, ...), but is unable to follow it through.

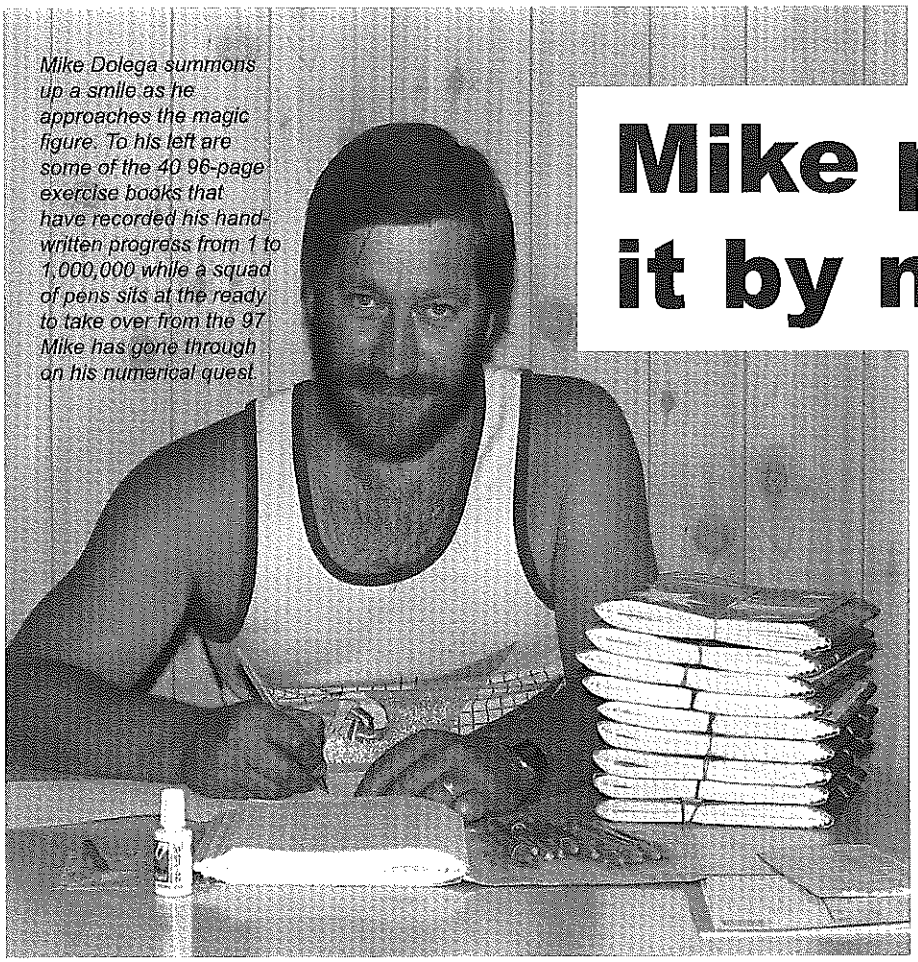
1 

Little progress

- Indicates an understanding of the requirements of the problem, but makes very limited progress on it.

TASK 15 SHEET

Mike and His Numbers



Mike Dolega summons up a smile as he approaches the magic figure. To his left are some of the 40 96-page exercise books that have recorded his hand-written progress from 1 to 1,000,000 while a squad of pens sits at the ready to take over from the 97 Mike has gone through on his numerical quest.

The photo is reproduced with permission from *The Advocate* newspaper, Burnie, Tasmania.

The Advocate

32 PAGES MONDAY FEBRUARY [illegible] 1989 40 CENTS

Mike playing it by numbers

By Angela Pilgrim

D'port man's got a million of 'em, and they're all handwritten

Mike Dolega has had the strangest obsession with numbers – but it is out of his system now.

For the past two years, Mike (33), of Devonport, has hand-written all the digits from one to a million, finishing yesterday.

Even Mike doesn't know how to identify his pasttime – it is hobby, habit and fascination with numbers rolled into one.

It all began two years ago when Mike was watching the Commonwealth Games on TV.

"It was late at night, and there were only a few events I was interested in," he said.

"So I just picked up a piece of paper and a pen and started writing – one, two, three..."

Yesterday afternoon, Mike reached one million.

His wife Ruby, who he says has been most tolerant for the past two years, counted down the final ten.

Mike says a million will do, and has no other figure-writing landmarks to chase.

He has written to the Guiness Book of Records to see if handwriting numbers to a million has been done before.

To reach the magic figure has taken 40 96-page exercise books, at a cost of $1 each.

Mike wrote the numbers down 20 columns across double pages, and there are 26 lines to a page.

He has used 97 ball point pens, costing a total of $66.50.

If all the figures Mike has written over the past two years were placed side-by-side, it would stretch for 176.36 km.

The total number of single digits Mike has written is 5,878,936.

It has taken 1292 hours to write to one million.

If he worked on it for eight hours a day, seven days a week, it would have taken 26.3 weeks.

Mike says writing the figures has given him an appreciation for how many a million really is.

If he were to write to one billion (a thousand million) at the rate it took to write the first million, it would take 2500 years!

His number writing has not been a waste of time. He can tell you what brands of pen last, and which ones blotch.

He will now take up a more conventional time filler – jigsaws.

He starts his first 3000-piece puzzle today.

TASK 16 SHEET

Multi Lotto

Game: The game uses all of the 100 flash cards with the basic multiplication facts. Players get to make their own boards by writing 16 different numbers into the squares of a 4 x 4 grid. As each flash card is shown, players cross that product on their game board. To win, a player must cross off all numbers on any row, column, diagonal, or all four corners of the board.

Multi Lotto game board

Felipe, Joe and Susan made the following game boards

12	5	29	36
28	46	87	50
81	54	14	8
63	10	7	35

FELIPE

1	2	3	4
20	44	90	79
18	25	9	10
37	36	35	34

JOE

16	9	18	24
5	21	0	30
14	48	72	45
12	33	17	20

SUSAN

1. Explain what you think each student's chances of winning might be and why.
2. Design a board you think would have a good chance of winning and give your reasons.

TASK 17 SHEET

Personalised Number Plates

You are working for the government selling personalised number plates.
They must use exactly 4 letters and the only ones available are

N A L E

1. How many four letter plates can you make from the above four letters?
   Show your reasoning.

2. Using what you have learnt so far, how many five letter plates are possible using the following letters?

P U M O R

Cards